Hillside Township School District

Mathematics Department
Calculus

Grades 12

Dr. Antoine L. Gayles
Superintendent of Hillside Public Schools

Dr. Christy Oliver-Hawley
Director of Curriculum and Instruction

Curriculum Contributors:
Mr. Emenaka, Mr. Thomas

Supervisor
Obinna Emenaka

Board of Education Approved:
January 19, 2017

Table of Contents

District Mission Statement

The mission of the Hillside Public Schools is to ensure that all students at all grade levels achieve the New Jersey Core Curriculum Content Standards and make connections to real-world success. We are committed to strong parent-community school partnerships, providing a safe, engaging, and effective learning environment, and supporting a comprehensive system of academic and developmental support that meets the unique needs of each individual.

Academic Area Overview

The Hillside Township School District is committed to excellence. We believe that all children are entitled to an education that will equip them to become productive citizens of the twenty-first century. We believe that a strong foundation in mathematics provides our students with the necessary skills to become competent problem solvers and pursue math intensive careers in the sciences and engineering.

A strong foundation in mathematics is grounded in exploration and rigor. Children are actively engaged in learning as they model real-world situations to construct their own knowledge of how math principles can be applied to solve every day problems. They have ample opportunities to manipulate materials in ways that are developmentally appropriate to their age. They work in an environment that encourages them to take risks, think critically, and make models, note patterns and anomalies in those patterns. Children are encouraged to ask questions and engage in dialogue that will lead to uncovering the math that is being learned. Facts and procedures are important to the study of mathematics. In addition to learning the common facts and procedures that lead efficient solutions of math problems, children will also have the opportunity to explore the "why" so that they can begin to understand that math is relevant to the world.

Our program provides teachers with resources both online and in print that incorporates the use of technology to help students reach the level of rigor that is outlined in the Common Core State Standards for Mathematics. Textbooks and materials have been aligned to the standards and teachers are trained on an ongoing basis to utilize the resources effectively and to implement research-based strategies in the classroom.

Affirmative Action Statement
Equality and Equity in Curriculum

The Hillside Township School District ensures that the district's curriculum and instruction are aligned to the State's Core Curriculum Content Standards and addresses the elimination of discrimination and the achievement gap, as identified by underperforming school-level AYP reports for State assessment, by providing equity in educational programs and by providing opportunities for students to interact positively with others regardless of race, creed, color, national origin, ancestry, age, marital status, affectional or sexual orientation, gender, religion, disability or socioeconomic status.

N.J.A.C. 6A:7-1.7(b): Section 504, Rehabilitation Act of 1973; N.J.S.A. 10:5; Title IX, Education Amendments of 1972

Math Department Lesson Plan Template

Lesson Information

Lesson Name: ___________________________
Unit: ___________________________________
Date: ___________________________________

Lesson Data

1. Essential Questions & Enduring Understanding:
2. CCSS:
3. Knowledge:
4. Skills:

5. Informal/Formal Assessment of Student Learning:

6. Lesson Agenda:

7. Homework:

UNIT 0: **Calculus Prerequisites**

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
✓ Equations and inequalities can describe, explain, and predict various aspects of the real world. ✓ Change in Algebra can be represented using slope as the ratio of vertical change to horizontal change. ✓ A line on a graph can be represented by a linear equation. ✓ A function that models a real world situation can be used to make predictions about future occurrences. ✓ Functions can be represented in a variety of ways such as graphs, tables, equations, or words. ✓ Defining trigonometric functions based on the unit circle provides a means of addressing situations that cannot be modeled by right triangles.	✓ How can we use equations or inequalities to model the world around us? ✓ How can we represent rates of change algebraically? ✓ Why is it necessary to have multiple ways of writing linear equations? ✓ How can we determine the appropriate function to use to model a situation? ✓ What are the advantages of having various representations of functions? ✓ Why would it be beneficial to define a unit of measure for angles that is independent of triangles?

CCSS	KNOWLEDGE	SKILLS
Equation solving, Slope, Linear Equations & Inequalities	**Students will know that**: • In solving linear equations or inequalities, the properties of real numbers can be used to simplify the original problem and ensure that the truth of the equation/inequality is maintained from the previous step to the next step in the process. • Factoring can be used to solve more complex equations and inequalities.	**Students will be able to**: • Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. • Solve equations requiring factoring, completing the square, or the quadratic formula.

	• The slope is a constant rate of change that measures the steepness of a line. - The higher the slope, the steeper the line. o $\text{slope } (m) = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$	• Calculate the slope of a line using the formula or by analyzing the graph of a linear function. • Calculate and interpret the average rate of change of a function, represented symbolically or numerically, over a specified interval. • Estimate the rate of change from a graph.
	• Linear equations can be written in the following forms: o Slope-Intercept Form: $y = mx + b$ o Point-Slope Form: $y - y_1 = m(x - x_1)$ or $y = m(x - x_1) + y_1$ o Standard Form: $Ax + By = C$	• Create equations in two variables to represent relationships between quantities. • Graph equations on coordinate axes (with labels & scales).
Functions, Graphs and Operations	**Students will know that:** • The following functions can be represented graphically, numerically, and algebraically: o Linear o Absolute value o Polynomial o Rational o Exponential & Logarithmic o Trigonometric & Inverse Trigonometric o Piecewise-Defined o Parametric • The properties of the graphs of functions are: o Domain/Range o Odd/Even o Symmetry o Zeros/Intercepts o Periodicity	**Students will be able to:** • Graph functions and show key features of the graph, by hand in simple cases or using technology for more complicated cases.
	• The operations of addition, subtraction, multiplication, division, and composition can be applied to functions and may effect the domain of the resulting function.	• Add, subtract, multiply, divide and compose functions. • Identify the domain of the resulting function after an operation has

		been applied.
Right Triangle Trigonometry & The Unit Circle	**Students will know that:** ● By similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. ● If θ is the measure of any acute angle in a right triangle, then the following are the trigonometric ratios for that angle: o $\sin\theta = \dfrac{Opposite}{Hypotenuse}$ o $\cos\theta = \dfrac{Adjacent}{Hypotenuse}$ o $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ ● The other three trigonometric functions, $\csc\theta,\ \sec\theta,\text{ and }\cot\theta$, are reciprocals of the basic three trigonometric functions above.	**Students will be able to:** ● Identify the opposite side, adjacent side, and the hypotenuse of a right triangle with respect to a given acute angle.
	● The ratios of the side lengths in the two special right triangles are: o For a $45^o - 45^o - 90^o$ triangle, the ratio is $1:1:\sqrt{2}$ o For a $30^o - 60^o - 90^o$ triangle, the ratio is $1:\sqrt{3}:2$	● Use the knowledge of ratios in special right triangles to compute the value of trigonometric functions for the acute angles, $30^o, 45^o,\text{ and } 60^o$ and their multiples.
	● The <u>unit circle</u> can be used to determine the values of all trigonometric functions. ● The common radian measures are: o $\theta = 0, \dfrac{\pi}{6}, \dfrac{\pi}{4}, \dfrac{\pi}{3},\text{ and } \dfrac{\pi}{2}$	● Recognize the degree equivalents for all common radian measures. ● Use the unit circle to compute the exact values of all six trigonometric function at the common radian measures and their multiples (without the use of a calculator).

	• The inverse of a trigonometric function is called an inverse trigonometric function.
	  o Inverse trigonometric functions can be used to determine the angle given the value of the ratio.
	  o Inverse trigonometric functions are important in various applications (i.e. angle of elevation/depression, uniform motion, etc…)
		• Calculate the angle measure that corresponds to the value of a trigonometric function using its inverse trigonometric function.

	Critical Vocabulary: Real Numbers, Slope, Forms of Equations (Slope-Intercept, Point-Slope , Standard), Linear, Absolute Value, Polynomial, Rational, Exponential, Logarithmic, Trigonometric, Inverse Trigonometric, Piecewise-Defined, Parametric, Domain, Range, Odd, Even, Symmetry, Zero, Intercepts, Periodic, Sine, Cosine, Tangent, Secant, Cosecant, Cotangent, Unit Circle, Radian, Composition, Hypotenuse.	
Summer Pre-Calculus Packet (To be completed prior to the start of Calculus course in September)		

UNIT 1: <u>Functions, Graphs, and Limits</u>

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
✓ With the aid of technology, graphs of functions are often easily produced. The emphasis shifts to the interplay between the geometric and analytical information and on the use of calculus to predict and explain the observed behavior of a function.	✓ How does an understanding of limits in this unit help to analyze functions graphically and analytically? ✓ How do the tools of calculus help to explain the behavior displayed by functions graphed with the aid of technology?
✓ The limit of a function can exist and be calculated even though a function fails to exist at a specified location.	✓ How is it possible that a function fails to exist at a point yet has a limit at that same point? ✓ What is the difference between the value of a limit and the value of a function?
✓ Limits can be used to describe unbounded behavior in functions.	✓ Why do some functions exhibit unbounded growth whereas others do not? How does calculus explain this behavior?
✓ Continuity is fundamental to calculus and ensures that every point along a graph of a function exists.	✓ Why is it necessary for both the function and its limit to exist at a given point to ensure that the function is continuous there? ✓ Why is continuity important in the study of calculus?

CCSS	KNOWLEDGE	SKILLS
Limits	**Students will know that**: • The <u>Limit</u> of a function is the value that the function approaches as the independent variable approaches a specified number. In many cases, this value is obvious, but sometimes is not. • The limit of $f(x)$ as x approaches some number, c is the value L. This is denoted by the notation: o $\lim_{x \to c} f(x) = L$	**Students will be able to**: • Calculate limits using algebra. • Estimate limits from graphs or tables of data.

	▪ The limit of a function may exist at $x=c$ even though the actual function does not exist at $x=c$. ▪ The limit of a function exists if and only if the limits from both sides of c exist and are equal. ▪ $\lim_{x\to c} f(x)=L$ exists if and only if $\lim_{x\to c^+} f(x)=\lim_{x\to c^-} f(x)=L$	
	● The properties of limits can be used to simplify expressions and determine the value of limits algebraically. o The limit of a constant is $\lim_{x\to c} k=k$ o The limit of the identity function is $\lim_{x\to c} x=c$ o *The properties of limits include*: the limit of a constant, sum, difference, product, quotient, constant multiple, and power. o *For polynomial functions:* $\lim_{x\to c} f\left(x\right)=f\left(c\right)$ o *For rational functions:* $\lim_{x\to c} f\left(x\right)=f\left(c\right)$, provided that the denominator is not zero at $x=c$.	● Use the properties of limits to simplify expressions and evaluate limits.
	● The following is a commonly known limit: o $\lim_{x\to 0}\frac{\sin x}{x}=1$	● Justify the value of $\lim_{x\to 0}\frac{\sin x}{x}$ using a graphing calculator. o Use this common limit to simplify expressions and evaluate limits algebraically.

Asymptotic and Unbounded Behavior	**Students will know that:**
	
	• Unbounded behavior created by the existence of asymptotes can be recognized graphically.
	
	• A horizontal asymptote exists when either of the following is true:
	  o $\lim_{x\to+\infty} f(x)=b$ or $\lim_{x\to-\infty} f(x)=b$, where b is some finite, real number.
	  o The line $y=b$ is the called a horizontal asymptote of $f(x)$.
	• A vertical asymptote exists when either of the following is true:
	  o $\lim_{x\to c^+} f(x)=\pm\infty$ or $\lim_{x\to c^-} f(x)=\pm\infty$
	  o The line $x=c$ is called a vertical asymptote of $f(x)$.
		Students will be able to:
		
		• Recognize unbounded behavior on a graph of a function that has asymptotes.
		
		• Describing asymptotic behavior in terms of limits involving infinity.
		• Comparing relative magnitudes of functions and their rates of change
		  o For example: *Contrasting exponential growth, polynomial growth, and logarithmic growth*.

	• The following is a commonly known limit:
	  o $\lim_{x\to\infty}\frac{\sin x}{x}=0$
		• Justify the value of $\lim_{x\to\infty}\frac{\sin x}{x}$ using a graphing calculator and/or the Squeeze (Sandwich) Theorem.
		  o Use this common limit to simplify expressions and evaluate limits algebraically.

Continuity	**Students will know that:** • A function is continuous if its graph can be drawn from beginning to end without lifting the pencil from the paper. o Graphs of continuous functions have no breaks or holes.	**Students will be able to:** • Determine whether or not a function is continuous intuitively. •

	• A function is continuous at any point, c in the interior of the function if both the function and its limit exist $x=c$ and are equal in value there. o $\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x) = f(c)$ • A functions is continuous at an endpoint on an interval $[a,b]$ if it is right continuous at a and left continuous at b. o Continuous at $x=a$ if $\lim_{x\to a^+} f(x) = f(a)$ o Continuous at $x=b$ if $\lim_{x\to b^-} f(x) = f(b)$	• Use their understanding of limits to evaluate the continuity of function at a point algebraically. • Use the definition of continuity at an endpoint to evaluate the continuity of functions on a given interval.
	• For a continuous function on the interval $[a,b]$, the Intermediate Value Theorem guarantees that the function has a value corresponding to every number along the interval.	• Check the validity of the Intermediate Value Theorem intuitively.
	<u>Critical Vocabulary</u>: Limit, One-Sided Limit, Infinity, Unbounded Behavior, Asymptote, Relative Magnitude, Continuity, Interval, Endpoint, Intermediate Value Theorem.	
Practice AP Test (Timed) – Unit 1		

Pacing Chart
UNIT 1: Functions, Graphs, and Limits

TIME FRAME	TOPIC	SUGGESTED PERFORMANCE TASKS ACTIVITIES/PROJECTS ASSESSMENTS	RESOURCES/INTERDISCIPLINARY CONNECTIONS
Early Sept – Mid Sept	Limits	Discusses the existence of limits in an interactive way http://www.calculus-help.com/storage/flash/limit02.swf Youtube video that explains the properties of limits http://www.youtube.com/watch?v=6webTCd5gEQ PowerPoint on limits and their properties http://nazmath.net/Calculus/Limits/Limits.ppt	Text Sections: 2.1 Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com
Mid Sept – End of Sept.	Asymptotic and Unbounded Behavior	Website provides basic info on asymptotes http://www.uoguelph.ca/numeracy/lofiles/asymptotes_final.swf Overview of asymptotes http://maretbccalculus2007-2008.pbworks.com/w/page/20301392/Describing%20asymptotic%20behavior%20in%20terms%20of%20limits%20involving%20infinity	Text Sections: 2.2 Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com

Last week of Sept – 1st week of Oct.	Continuity	Discusses continuity and the importance of continuous functions in calculus http://www.calculus-help.com/continuity/ PowerPoint overview of continuity www.online.math.uh.edu/HoustonACT/...**Calculus**.../Calc02_3.ppt	Text Sections: 2.3, Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com

UNIT 2: Derivatives

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
✓ Calculus provides tools that extend beyond the limitations of algebra and can be used to define change at an instant in time. ✓ Derivative shortcuts simplify the algebra in calculus allowing for more time to be spent on interpreting the meaning of the results. ✓ The derivative can be used to analyze graphs of functions revealing critical information that can be visualized using technology. ✓ Differential equations arise in the study of engineering and science. An understanding of the relationship between derivatives and antiderivatives is necessary when solving such equations.	✓ Since slope is the rate of change, why can't it be used to represent rate of change at an instant? ✓ What is the difference between average rate of change and instantaneous rate of change? ✓ Is it necessary to evaluate the difference quotient each time the derivative needs to be determined? ✓ Knowing that the derivative represents the slope at a point on a curve, how can this be used to predict various behaviors in graphs of functions? ✓ How can differential equations be used to model situations and solve complex problems?

CCSS	KNOWLEDGE	SKILLS

Instantaneous Rate of Change and the Derivative	**Students will know that**:	**Students will be able to**:

	• The average rate of change and the instantaneous rate of change are <u>not</u> the same.
	  o The average rate of change is the slope of the line passing through two points of the graph.
	  o The instantaneous rate of change can be thought of as the slope of a line passing through a single point (tangent line) on the graph. This line approximates the slope of the curve at that point.
	  o The instantaneous rate of change is the limit of the average rate of change.
	  o The instantaneous rate of change can be approximated by zooming in on a point to observe local linearity.
		• Approximate the instantaneous rate of change from graphs and tables of values using graphing technology.
		• Compare the average rate of change and the instantaneous rate of change given a set of data.
		  o For example: *Data on an a falling object can be graphed and used to compute both types of rates of change*.
		• Interpret the derivative as a rate of change in context.
		  o Including: *velocity, speed, and acceleration*.

	• The derivative can be interpreted as the instantaneous rate.
	  o The derivative can also be interpreted as the slope of the tangent line to a curve at a point.
	  o Derivative can be represented algebraically as the limit of the difference quotient:
	    ▪ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
		• Approximate the derivative from graphs and tables of values using graphing technology.
		• Evaluate the derivative of a function algebraically by evaluating the difference quotient.
		• Determine the equation of the tangent line at a point.

	• The relationship between differentiability and continuity is:
	  o If a function is differentiable at a point then, it must be continuous there; however continuity does not guarantee differentiability.
	    ▪ *For example*: $f(x)=\|x\|$ is continuous everywhere, but not differentiable at $x=0$.
	  o A function whose graph is otherwise smooth will fail to have a derivative at a point where there is a: *corner, cusp, vertical tangent, or discontinuity.*
		• Recognize the difference between differentiability and continuity.
		• Determine whether or not a function is differentiable at a point by looking its graph.

<table>
<tr><td rowspan="2">Differentiation</td><td>Students will know that:
Evaluating the difference quotient to determine every derivative can be tedious and time-consuming. The rules of differentiation are used do this more efficiently.</td><td>Students will be able to:
Use the rules of derivatives to differentiate various functions:
Differentiate a constant, power, constant multiple, sum, difference, product, and quotient.
Differentiate common functions including: exponential, logarithmic, trigonometric, and inverse trigonometric.</td></tr>
<tr><td>Some problems involve more than one changing quantity. The rates of change of these quantities can be linked mathematically by implicit differentiation.</td><td>Use more advance differentiation techniques to determine the derivative of complex functions: Chain Rule & Implicit Differentiation
Use implicit differentiation to find the derivative of an inverse function.
Model and solve related rates problems.</td></tr>
<tr><td>Analytical Techniques</td><td>Students will know that:
The Extreme Value Theorem states that if a function is continuous on a closed interval, then it must have a maximum and a minimum value on the interval.
Relative and absolute extrema can be determined graphically, numerically, and algebraically.
Relative extrema of a function only occur at critical points.
Critical points are values on the interval, $[a,b]$ that are either of the following:
Endpoints
Points where $f'(x)=0$
Points where $f'(x)$ does not exist.</td><td>Students will be able to:
Identify maxima and minima for a function graphically, numerically, and algebraically.
Set up and solve optimization problems.</td></tr>
</table>

	• The derivative can be used to analyze graphs of a function. o If $f'(x) > 0$ for all values of the domain on an interval, then $f(x)$ is increasing on that interval. o If $f'(x) < 0$ for all values of the domain on an interval, then $f(x)$ is decreasing on that interval. o Changes in monotonicity occur at extreme values.	• Determine the intervals on a domain for which a function is increasing or decreasing algebraically. • Recognize increasing and decreasing behavior in graphs of functions.
	• The Mean Value Theorem states: If a function is continuous on a closed interval, then at some point in this interval the instantaneous rate of change will equal the average rate of change. o $f'(c) = \frac{f(b)-f(a)}{b-a}$, for some value of "$c$" on the interval $[a,b]$. o This theorem can be used to prove many of the theorems in this unit.	• Check the validity of the Mean Value Theorem intuitively. • Determine if a function satisfies the Mean Value Theorem on a given interval.
	• The second derivative can be used to analyze the graph of a function and the graph of its derivative. o If $f''(x) > 0$ for all values of the domain on an interval then $f(x)$ is concave up (derivative increasing). o If $f''(x) < 0$ for all values of the domain on an interval then $f(x)$ is concave down (derivative decreasing). o Points of inflection are places where concavity changes.	• Determine the intervals on a domain for which a function is concave up or down algebraically. • Recognize concavity in graphs of functions as it relates to the behavior of the first derivative. • For example: Observe concave up behavior on an interval as increasing tangent line slopes as one moves from left to right.

Anti-Differentiation & Differential Equations	**Students will know that:** • The inverse of differentiation is called antidifferentiation. o $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$ ▪ Functions with the same derivative differ by a constant. The true antiderivative of $f(x)$ is $F(x)+C$, where C is any constant. o The rules for finding derivatives can be used in reverse to determine the antiderivatives. o The notation for antiderivatives is: $\int f(x)\, dx = F(x) + C$	**Students will be able to:** • Determine the antiderivative of a variety of functions using an understanding of derivatives as the inverse of antiderivatives. • Use the rules of antidifferentiation to determine the antiderivative of various functions o Determine the antiderivative of: *a constant, power, constant multiple, sum, and difference.* o Find the antiderivative of common functions including*: exponential, logarithmic, trigonometric, and inverse trigonometric*.
	• An equation that involves derivatives and has a function as its solution is called a differential equation. o The order of the differential equation is the order of the highest derivative in the equation. o Several techniques exist for solving differential equations of varying complexity.	• Translate a verbal description of a problem into a differential equation that can be solved mathematically. • Solve simple differential equations. o Antidifferentiation o Solve separable differential equations and use them to model various applications (i.e. The equations $y' = ky$ can be used to model exponential growth). o Find specific solutions to differential equations using the initial conditions.
	• A slope field is a visual representation of all possible solutions to a differential equation and is determined by evaluating the slope of the curve at various locations on the coordinate plane. • A slope field can be used to approximate the solution curve that passes through various points	• Create a slope field given a differential equation. • Recognize which slope field belongs to a given differential equation by evaluating the equation at various points and using an understanding of slope.
	Critical Vocabulary: Average Rate of Change, Instantaneous Rate of Change, Tangent Line, Derivative, Difference Quotient, Differentiability, Differentiation, Corner, Cusp, Vertical Tangent, Discontinuity, Chain Rule, Implicit Differentiation, Related Rates, Extreme Value Theorem, Maximum, Minimum, Relative Extrema, Absolute Extrema, Critical Points, Interval, Optimization, Monotonicity, Concavity, Mean Value Theorem, Inflection Point, Antiderivative, Antidifferentiation, Differential Equation, Slope Field	
Practice AP Test (Timed) – Unit 2		

Pacing Chart
UNIT 2:

TIME FRAME	**TOPIC**	**SUGGESTED PERFORMANCE TASKS ACTIVITIES/PROJECTS ASSESSMENTS**	**RESOURCES/INTERDISCIPLINARY CONNECTIONS**
Early Oct. – Mid Oct.	Instantaneous Rate of Change and the Derivative	This websites addresses graphing of the tangent line, the derivative, and the area under a curve for functions. http://illuminations.nctm.org/ActivityDetail.aspx?ID=221 Discusses the connection between average and instantaneous rate of change. http://www.google.com/url?sa=t&rct=j&q=instantaneous+rate+of+change+filetype:ppt&source=web&cd=1&cad=rja&ved=0CD8QFjAA&url=http%3A%2F%2Ffacstaff.elon.edu%2Famancuso2%2Fmth116%2FInstantaneous%2520Rate%2520of%2520Change.ppt&ei=kVYyUPzePJSH0QGqk4DQBA&usg=AFQjCNH6Dh25hhB-75CtynyvI13AYCEwPA This flash file consists of teacher demonstrating concept of instantaneous rate of change and basic derivative rules. Also real life situations are addressed to reflect each topic. http://www.montereyinstitute.org/courses/General%20Calculus%20I/course%20files/multimedia/unit3intro/3_00_01.swf	Text Sections: 2.4, 3.1, 3.2, 3.4 Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com

Mid Oct. – End of Nov.	Differentiation	This websites addresses graphing of the tangent line, the derivative, and the area under a curve for functions. http://illuminations.nctm.org/ActivityDetail.aspx?ID=221 This flash file consists of teacher demonstrating concept of instantaneous rate of change and basic derivative rules. Also real life situations are addressed to reflect each topic. http://www.montereyinstitute.org/courses/General%20Calculus%20I/course%20files/multimedia/unit3intro/3_00_01.swf	Text Sections: 3.3, 3.5, 3.6, 3.7, 3.8, 3.9, 4.6, Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com.
Early Dec. – Mid Jan.	Analytical Techniques	This link addresses the concept of the Mean Value Theorem for Derivatives. This is a graphing calculator demo that addresses the overall meaning of the theorem. http://education.ti.com/xchange/US/Math/Calculus/16074/02%20MVTForDerivatives.swf Overview of Optimization http://academic.brcc.edu/ryanl/modules/multivariable/differentiation/optimization/optimization_pt1.swf This is an overview that addresses the extreme value theorem and talks about finding critical values. Also, the process of finding absolute extrema is expressed visually. http://www.scs.sk.ca/hch/harbidge/Calculus%2030/Unit%205/lesson%202/maxmin.swf	Text Sections: 4.1, 4.2, 4.3, 4.4, Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com
End of Jan. – Early Feb.	Anti-Differentiation & Differential Equations	Discusses the connection between derivatives and anti-derivatives. The site also addresses solving differential equations. http://uccpbank.k12hsn.org/courses/APCalculusABII/course%20files/multimedia/unit5intro/5_00_01.swf This link has a demo for how to sketch the slope field for differential equations. http://archives.math.utk.edu/visual.calculus/7/fields.1/movie1.swf Discusses separable differential equations http://s3.amazonaws.com/cramster-resource/34203_Calc06_2.swf	Text Sections: 6.2 (Properties only pg. 332), 6.1, 6.4, Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com

		Applications of separable differential equations http://www.google.com/url?sa=t&rct=j&q=separable%20differential%20equations%20filetype%3Appt&source=web&cd=4&ved=0CEQQFjAD&url=http%3A%2F%2Fmath.kennesaw.edu%2F~sellerme%2Fsfehtml%2Fclasses%2Fmath2202%2Fseparabledeapplications.ppt&ei=m3QyUMTSDqP50gHd64GgBA&usg=AFQjCNGLU7gEdKrYtF3muEgUlqZ-zHt7iw	

UNIT 3: Integrals

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
✓ The area of an irregularly, shaped object can be approximated using rectangles for which the area can be easily computed. The larger the number of objects the more precise the approximation. ✓ The Fundamental Theorem of Calculus links differentiation and integration while providing powerful tools for evaluating integrals analytically. ✓ Numerical approximations, Riemann sums, and the definite integral have many applications from diverse disciplines.	✓ How is it possible for the area of an infinite number of rectangles to be finite? ✓ Why is the Fundamental Theorem of Calculus so important to the subject? ✓ When is it possible to set up a Riemann Sum as an approximation for area?

CCSS	KNOWLEDGE	SKILLS
Definite Integral and Properties	**Students will know that**: ● Definite integrals can be represented as a limit of Riemann sums o An integral can be interpreted as the sum of infinitely small quantities. This sum has a limiting value that can be computed with algebra. ▪ *For example: rectangular approximations for the area under a curve become exact as the width of the rectangles used approaches zero.* o The integral can also be interpreted as the limiting value of the sum of an infinite number of quantities. ▪ *For example: rectangular approximations for the area under a curve become exact when the number of rectangles is infinite.*	**Students will be able to**: ● Set up and compute Riemann sums for a variety of problems. o Area under a curve ● Use numerical approximations to evaluate definite integrals o Make Riemann sum approximations using left, right, or midpoint as the evaluation points. o Use trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically and by tables of values.

	• The Fundamental Theorem of Calculus relates the definite integral of a function to its antiderivative denoted by the following equality: o $\int_a^b f\left(x\right)dx = F\left(b\right) - F\left(a\right)$ o Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval. ▪ $\int_a^b f'(x)dx = f(b) - f(a)$	• Use of the Fundamental Theorem to: o Represent a particular antiderivative, and the analytical and graphical analysis of functions so defined. o Evaluate definite integrals.
	• Basic properties of definite integrals (examples include additivity and linearity). o The derivative of a definite integral ▪ $\frac{d}{dx}\int_a^x f\left(t\right)\,dt = f\left(x\right)$	• Use the properties of definite integrals and knowledge of basic antiderivatives to evaluate simple integrals.
Advance Integration Techniques & Applications	**Students will know that:** • Antiderivatives can be evaluated by changing the variable of integration using u-substitution resulting in a simpler expression to integrate. o For definite integrals, the new limits of integration are functions of the substituted variable and must be changing accordingly.	**Students will be able to:** • Perform u-substitution on both indefinite and definite integrals and evaluate, changing limits of integration when appropriate.
	• Whatever applications are chosen, the emphasis is on using the method of setting up an approximation Riemann sum and representing its limit as a definite integral.	• Set up an approximation Riemann sum and represent its limit as a definite integral in the following applications: o Finding the area of a region (Under curve, between curves, etc…) o Volume of a solid with known cross sections (Volumes of revolution) o Average value of a function o Distance traveled by a particle along a line o Accumulated change from a rate of change.

	Critical Vocabulary: Definite Integral, Riemann Sum, Integral, Rectangular Approximation, Numerical Approximation, Trapezoidal Sum, Fundamental Theorem of Calculus, U-Substitution,
Practice AP Test (Timed) – Unit 3	

Pacing Chart
UNIT 3:

TIME FRAME	TOPIC	SUGGESTED PERFORMANCE TASKS ACTIVITIES/PROJECTS ASSESSMENTS	RESOURCES/INTERDISCIPLINARY CONNECTIONS
Mid. Feb – Early Mar.	Definite Integral and Properties	This websites addresses graphing of the tangent line, the derivative, and the area under a curve for functions. http://illuminations.nctm.org/ActivityDetail.aspx?ID=221 This link is a visual demo of using the fundamental theorem of calculus. This is a graphing calculator application. http://education.ti.com/xchange/US/Math/Calculus/16113/06%20Area%20Function%20Demonstration.swf	Text Sections: 5.1, 5.2, 5.3, 5.4, 5.5, Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com
Mid Mar. – Early Apr.	Advance Integration Techniques & Applications	This link addresses the process of u-substitution for indefinite and definite integrals. http://wps.prenhall.com/wps/media/objects/426/436914/uSubs3.swf This link is a graphing calculator demo that illustrates the area under a curve using various types of functions. http://education.ti.com/xchange/US/Math/Calculus/16114/06%20AreaFunctionProblems.swf This link shows a careful, colorful step-by-step procedure on drawing solids of revolution. This is a visual approach to the washer method. http://www.jeffsims.net/flash/revolution5.swf This link shows the visual process of finding the area of regions between two curves. http://mcs.mscd.edu/movies/Areas_Between_Curves.swf	Text Sections: 6.2, 7.1, 7.2, 7.3,7.4, 7.5 Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com