Hillside Township School District

Mathematics Department
Trig. Functions & Statistics CP

Grades 11

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Board of Education Approved:
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District Mission Statement

The mission of the Hillside Public Schools is to ensure that all students at all grade levels achieve the New Jersey Core Curriculum Content Standards and make connections to real-world success. We are committed to strong parent-community school partnerships, providing a safe, engaging, and effective learning environment, and supporting a comprehensive system of academic and developmental support that meets the unique needs of each individual.

Academic Area Overview

The Hillside Township School District is committed to excellence. We believe that all children are entitled to an education that will equip them to become productive citizens of the twenty-first century. We believe that a strong foundation in mathematics provides our students with the necessary skills to become competent problem solvers and pursue math intensive careers in the sciences and engineering.

A strong foundation in mathematics is grounded in exploration and rigor. Children are actively engaged in learning as they model real-world situations to construct their own knowledge of how math principles can be applied to solve every day problems. They have ample opportunities to manipulate materials in ways that are developmentally appropriate to their age. They work in an environment that encourages them to take risks, think critically, and make models, note patterns and anomalies in those patterns. Children are encouraged to ask questions and engage in dialogue that will lead to uncovering the math that is being learned. Facts and procedures are important to the study of mathematics. In addition to learning the common facts and procedures that lead efficient solutions of math problems, children will also have the opportunity to explore the "why" so that they can begin to understand that math is relevant to the world.

Our program provides teachers with resources both online and in print that incorporates the use of technology to help students reach the level of rigor that is outlined in the Common Core State Standards for Mathematics. Textbooks and materials have been aligned to the standards and teachers are trained on an ongoing basis to utilize the resources effectively and to implement research-based strategies in the classroom.

Affirmative Action Statement
Equality and Equity in Curriculum

The Hillside Township School District ensures that the district's curriculum and instruction are aligned to the State's Core Curriculum Content Standards and addresses the elimination of discrimination and the achievement gap, as identified by underperforming school-level AYP reports for State assessment, by providing equity in educational programs and by providing opportunities for students to interact positively with others regardless of race, creed, color, national origin, ancestry, age, marital status, affectional or sexual orientation, gender, religion, disability or socioeconomic status.

N.J.A.C. 6A:7-1.7(b): Section 504, Rehabilitation Act of 1973; N.J.S.A. 10:5; Title IX, Education Amendments of 1972

Math Department Lesson Plan Template

Lesson Information

Lesson Name: _____
Unit: _____
Date: _____

Lesson Data

1. Essential Questions & Enduring Understanding:

2. CCSS:

3. Knowledge:

4. Skills:

5. Informal/Formal Assessment of Student Learning:

6. Lesson Agenda:

7. Homework:

UNIT 1: Trigonometric Functions

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
✓ An understanding of right triangles can aid in a variety of applications that may require indirect measurement.	✓ How can trigonometry be used to solve problems that involve indirect measurement and large distances?
✓ Defining trigonometric functions based on the unit circle provides a means of addressing situations that cannot be modeled with the tools of geometry.	✓ Why are radian measures used rather than degree measures in working with trigonometric functions and their applications?
✓ An understanding of coordinate geometry and functions are essential to the study of trigonometry.	✓ What prior knowledge is needed for understanding trigonometric functions and its uses?
✓ Trigonometric functions can be applied to areas of study involving periodic behavior.	✓ How can you model periodic behavior?

CCSS	KNOWLEDGE	SKILLS
Right Triangle Trigonometry G.SRT.6 G.SRT.7 G.SRT.8	**Students will know that**: ● The Pythagorean Theorem states that the sum of the square of each leg of a right triangle equals the square of the hypotenuse. o $a^2 + b^2 = c^2$ ● By similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. ● The following trigonometric ratios are: o $\sin\theta = \frac{Opposite}{Hypotenuse}$ and $\csc\theta = \frac{1}{\sin\theta}$ o $\cos\theta = \frac{Adjacent}{Hypotenuse}$ and $\sec\theta = \frac{1}{\cos\theta}$	**Students will be able to**: ● Use the Pythagorean Theorem along with any necessary trigonometric ratios to determine angles and side lengths in a right triangle as well as solve application problems. ● Explain and use the relationship between the sine and cosine of complementary angles. ● Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

	o $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{1}{\tan\theta}$	
Unit Circle, Trigonometric Functions & Their Graphs F.TF.1 F.TF.2 F.TF.5 F.TF.8	**Students will know that:** ● The <u>Unit Circle</u> is a circle with a radius of one and center located at the origin. o The unit circle can be used to represent various angle measures ● The <u>Radian</u> measure of an angle is the length of the arc on the unit circle subtended by the angle. o Arc length is defined by equation $S = \theta r$	**Students will be able to:** ● Explain how the unit circle in the coordinate plane: o Enables the extension of trigonometric functions to all real numbers o Can be used to interpret radian measures of angles created from rotations around the unit circle.
	● Points on the unit circle have an x-coordinate of $\cos\theta$ and a y-coordinate of $\sin\theta$. o Ordered pairs can be written as $(\cos\theta, \sin\theta)$ o The value of the cosine or sine can be positive or negative based on the quadrant. ● All six trigonometric functions can be computed for common angle measures without using a calculator (i.e. Trig function charts, special right triangles, unit circle)	● Use the unit circle to determine $\cos\theta$ and $\sin\theta$ for various angle measures. ● Determine the values of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$, and $\cot\theta$ for the following common angles and their multiples <u>without the use of a calculator</u>: o $\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$, and $\frac{\pi}{2}$ ● Use a trig functions chart of common angles and an understanding of the unit circle to evaluate all six trigonometric functions. ● Determine the value of $\sin\theta$, $\cos\theta$, or $\tan\theta$ given a value of any of the three trigonometric functions and a known quadrant.

	• An inverse trigonometric function can be used to compute the value of an angle measure given the value of a trigonometric function o $\sin^{-1}\theta,\ \cos^{-1}\theta,\ \tan^{-1}\theta$	• Find the angle measure given the value of a trigonometric function.
	• The Pythagorean Theorem can be used to prove trigonometric identities.	• Prove all three Pythagorean identity o $\sin^2\theta+\cos^2\theta=1$ o $1+\tan^2\theta=\sec^2\theta$ o $\cot^2\theta+1=\csc^2\theta$ • Use a Pythagorean Identity, a value for a trigonometric function, and a known quadrant to find any of the six trigonometric functions.
	• Trigonometric functions have an amplitude, frequency and midline. o $f(x)=A\sin(Bx+C)+D$ o $f(x)=A\cos(Bx+C)+D$ ▪ The absolute value of A is the amplitude. ▪ The frequency is the number of full cycles in an interval (2π). ▪ $frequency=\frac{B}{2\pi}$ ▪ Phase shift is horizontal shift in the graph and is determined by the value and sign of $\frac{C}{B}$. o The <u>Midline</u> is the horizontal line that passes through the middle of a trigonometric function. ▪ $y=\frac{Max+Min}{2}$	• Graph trigonometric functions, showing period, midline, and amplitude. • Choose a trigonometric function to model periodic behaviors with specified amplitude, frequency, and midline.

Extensions of Trigonometry Laws of Sines and Cosines G.SRT.9 G.SRT.10 G.SRT.11	**Students will know that:** ● The Law of Sines and the Law of Cosines can be used to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	**Students will be able to:** ● (+) Prove the Laws of Sines and Cosines and use them to solve problems. ● (+) Apply the Law of Sines and Cosines to find unknown measurements in both right and non-right triangles ● (+) Derive the formula $A = \frac{1}{2}ab\sin C$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
	● Addition and subtraction formulas for the cosine and sine are o $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\cos\beta$ o $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ ● Double angle formulas for the cosine and sine a o $\sin 2\theta = 2\sin\theta\cos\theta$ o $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$	● Find the exact values of sine and cosine of uncommon angles that are the sum or difference of common angles without using a calculator. ● Use the double angle formulas to prove trigonometric identities.
	Critical Vocabulary: Radian measure, Arc, Arc Length, Unit Circle, Trigonometric Functions, Inverse Trigonometric Functions, Real Numbers, Period, Periodic, Amplitude, Frequency, Midline, Pythagorean Identity, Sine, Cosine, Tangent, Law of Sines, Law of Cosines	
Common 1st Quarter Assessment		

Pacing Chart
UNIT 1: Trigonometric Functions

TIME FRAME	TOPIC	SUGGESTED PERFORMANCE TASKS ACTIVITIES/PROJECTS ASSESSMENTS	RESOURCES/INTERDISCIPLINARY CONNECTIONS
Beginning of School until the 3rd week of September	Right Triangle Trigonometry	Teacher Resources: Chapter 5 Project #4 Videos lessons of trigonometric concepts. http://wps.aw.com/aw_demana_precalculus_8/ Trigonometric relationships http://www.clarku.edu/~djoyce/trig/identities.html Supplemental practice problems in Trigonometry www.interactivemath.com Sample problems with solutions that show how to apply trigonometry for right triangles http://www.themathpage.com/atrig/solve-right-triangles.htm Interactive tool that explains the relationships in special right triangles http://www.mrperezonlinemathtutor.com/G/3_3_Using_30_60_90_and_45_45_90_ratios.html	Text Sections: 4-4, 5-1, 5-9 Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com
4th week of September until the end of October	Unit Circle, Trigonometric Functions & Their Graphs	Teacher Resources: Chapter 4 Project #1, #2, #5 Videos lessons of trigonometric concepts. http://wps.aw.com/aw_demana_precalculus_8/ Supplemental practice problems in Trigonometry www.interactivemath.com Information on the Law of Cosines http://en.wikipedia.org/wiki/Law_of_cosines Various tools for analysis of the unit circle & trigonometric identities http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCou	Text Sections: 4-2, 4-3, 4-4, 4-8, 5-2, 5-4, 5-6, 13-1, Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com

		rse&CourseID=254 Video on graphing trigonometric functions http://www.youtube.com/watch?v=80c_F0-7ZxE	
End of October until the first week of November	Extensions of Trigonometry Laws of Sines and Cosines	Teacher Resources: Chapter 5 Project #1, #5; Chapter 12 #5 Videos lessons of trigonometric concepts. http://wps.aw.com/aw_demana_precalculus_8/ Supplemental practice problems in Trigonometry www.interactivemath.com Various tools for analysis of trigonometric identities http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=254	Text Sections: 5-3, 5-5, 12-5, 12-6 Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com

UNIT 2: Polynomial, Rational, and Radical Relationships

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
✓ The solutions of polynomial equations can be extended to include the set of all complex numbers.	✓ Why are complex numbers important?
✓ An understanding of polynomial functions allows for generalization to be made that can be applied to previous studied functions.	✓ How does an understanding of operations on numbers help in better understanding more complex algebraic expressions or operations?
✓ Polynomial functions allow us to model real world applications found in various mathematical disciplines.	✓ When does it make sense to use a polynomial function to model a situation?
✓ An understanding of the mathematics behind fractions and polynomials can help in making sense of rational expressions.	✓ What are the similarities between working with basic fractions and working with rational expressions? Differences?
✓ Sometimes solving equations correctly results in answers that are incorrect and must be checked.	✓ Why are answers that are derived using correct methods actually incorrect?

CCSS	KNOWLEDGE	SKILLS
Polynomials, Complex Numbers & Operations N.CN.1 N.CN.2 N.CN.8 A.SSE.1 A.SSE2 A.APR.2 A.APR.4	**Students will know that:** ● There is a complex number *i* such that $i^2 = -1$, and every complex number has the form *a* + *bi* where *a* and *b* are real numbers. ● A Polynomial is an expression consisting of the sum of two or more terms each of which is the product of a constant and a variable raised to a whole number exponent.	**Students will be able to:** ● Use the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. ● Identify parts of an expression, such as terms, factors, and coefficients. ● Interpret the parts of an algebraic expression that represent a problem in context. o Apply this to complicated expressions by viewing one or more of their parts as a single entity. ▪ *For example, interpret* $A = P(1+r)^n$ *as the product of P and a factor not depending on P.*

A.APR.5 A.APR.6	• Polynomials form a system that is closed under the operations of addition, subtraction, and multiplication.	• Add, subtract, and multiply polynomials. • Use the structure of an expression to identify ways to rewrite it. o *For example, see* $x^4 - y^4$ *as* $\left(x^2\right)^2 - \left(y^2\right)^2$ *, thus recognizing it as a difference of squares that can be factored as* $\left(x^2 - y^2\right)\left(x^2 + y^2\right)$. • Prove polynomial identities and use them to describe numerical relationships. o For example, the polynomial identity $\left(x^2 + y^2\right)^2 = \left(x^2 - y^2\right)^2 + \left(2xy\right)^2$ can be used to generate Pythagorean triples. o (+) Extend polynomial identities to the complex numbers. ▪ *For example,* $x^2 + 4$ *can be written as* $\left(x + 2i\right)\left(x - 2i\right)$.
	• (+) The Binomial Theorem states that $\left(x + y\right)^n = P_0 x^n + P_1 x^{n-1} y + P_2 x^{n-2} y^2 + \ldots + P_{n-2} x^2 y^{n-2} + P_{n-1} x y^{n-1} + P_n y^n$ o (+) The coefficients $P_0, P_1, P_2, \ldots, P_n$ of the binomial expansion can be determined from the n^{th} row of Pascal's Triangle.	• (+) Apply the Binomial Theorem for the expansion of $\left(x + y\right)^n$ in powers of *x* and *y* for a positive integer *n*.
	• The Remainder Theorem states that the remainder of $\frac{p(x)}{x - a}$ is $p(a)$ o The Remainder Theorem only applies to situations involving division by a linear polynomial with a leading coefficient of 1. o *p(a)* = 0 if and only *if (x – a)* is a factor of p(x).	• Apply the Remainder Theorem for a polynomial. • Divide polynomials using inspection and long division. o Use a computer algebra system for more complicated division problems. • Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ *)* in the form $q(x) + \frac{r(x)}{b(x)}$, o *a(x), b(x), q(x),* and *r(x)* are polynomials.

		o The degree of *r(x)* is less than the degree of *b(x)*.
Solving Polynomial Equations N.CN.7 N.CN.9 A.APR.3	**Students will know that:** ● Completing the square, factoring, or the quadratic formula can be used to solve quadratic equations with complex solutions. ● (+) The Fundamental Theorem of Algebra states that the number of complex solutions to a polynomial equation is equal to the degree of the polynomial. ● The key features of a polynomial function can be observed on a graph.	**Students will be able to:** ● Solve quadratic equations with real coefficients that have complex solutions. ● Create quadratic equations and inequalities in one variable and use them to solve problems. ● (+) Show that The Fundamental Theorem of Algebra is true for quadratic polynomials. ● Graph functions and show key features of the graph, by hand in simple cases or use technology for more complicated cases. o Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior ● Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Rational & Radical Expressions and Their Equations	**Students will know that:** ● An understanding of the basic rules of exponents can be used to simplify expressions and solve radical equations. ● Radical equations may have extraneous solutions therefore all solutions must be checked by substituting into the original equation.	**Students will be able to:** ● Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A.SSE.4 A.APR.7 A.REI.2	• Rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression. • The formula for the sum of a finite geometric series is: o $S_n = \frac{g_1\left(r^n - 1\right)}{r-1}$	• Create equations in one variable and use them to solve problems involving simple rational functions. • Add, subtract, multiply, and divide rational expressions. • Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), o Use the formula to solve problems. ▪ *For example, calculate mortgage payments.*
	Critical Vocabulary: Real Numbers, Complex Numbers, Terms, Expression, Coefficient, Constant, Polynomial, Degree, Closed, Factor, Remainder, End Behavior, Equation, Zeros, Radical, Rational, Finite Geometric Series, Common Ratio, Extraneous Solution, closed under an operation,	
Common Midterm Exam:		

Pacing Chart
UNIT 2: Polynomial, Rational, and Radical Relationships

TIME FRAME	TOPIC	SUGGESTED PERFORMANCE TASKS ACTIVITIES/PROJECTS ASSESSMENTS	RESOURCES/INTERDISCIPLINARY CONNECTIONS
Early November until end of December	Polynomials, Complex Numbers & Operations	2 websites on operations on complex numbers http://www.purplemath.com/modules/complex.htm http://www.purplemath.com/modules/complex2.htm, Glencoe practice work on rational and radical expressions and equations www.math.glencoe.com Free resources Algebra resources www.kutasoftware.com Displays the graphs of many types of functions http://academic.pgcc.edu/~lwojciec/projects/proj104/Lines/relationship.swf Online calculator for the synthetic division of polynomial equations with a third degree. http://easycalculation.com/algebra/polynomial-long-division.php Polynomial Calculator representing the product and the division of the polynomials http://www.solvemymath.com/online_math_calculator/algebra_combinatorics/polynomial_calculator/polynomial_mul_div.php Polynomial Long Division lesson http://www.youtube.com/watch?v=XiZKGk8dbRc	Text Sections: 7-1, 7-2, 7-3, 7-5, 10-2, 10-3, Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com

Early January until the end of January	Solving Polynomial Equations	Teacher Resources: Chapter 7 Project #4 Solving quadratic equations by completing the square http://www.mathsisfun.com/algebra/completing-square.html Shows how to derive the quadratic formula by completing the square http://www.mathsisfun.com/algebra/quadratic-equation-derivation.html Glencoe practice work on rational and radical expressions and equations www.math.glencoe.com Free resources Algebra resources www.kutasoftware.com	Text Sections: 7-5, 7-6, 7-7, 7-8 Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com
Early February until middle of February	Rational & Radical Expressions and Their Equations	Step by step solutions to rational equations http://hotmath.com/help/gt/genericalg1/section_7_5.html Simplifying radicals and expressions with rational exponents. Also radical equations. http://www.slideshare.net/jessicagarcia62/simplifying-radical-expressions-rational-exponents-radical-equations Polynomials, Rational expressions, Radicals expressions http://mathworld.wolfram.com/ Various mathematical concepts addressed http://www.mathmusic.org/mathmatical-expressions/graphing-lines/solving-rational-expressions.html Geometric series http://www.purplemath.com/modules/series5.htm Proof of the geometric series formula http://fym.la.asu.edu/~tturner/MAT_117_online/SequenceAndSeries/Geometric_Sequences.htm Glencoe practice work on rational and radical expressions and equations www.math.glencoe.com Free resources Algebra resources www.kutasoftware.com	Text Sections: (Not Supported by current textbook) Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com

UNIT 3: Modeling with Functions

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
✓ Functions can be used to model relationships both real world and abstract. ✓ Different types of functions can be identified by their graphs. ✓ Exponential functions can be used to model situations involving rapid growth or decay. ✓ Changes to a function in algebraic form result in predictable changes to its graph. ✓ Systems of equations and inequalities allow us to organize information to make informed decisions and solve complex problems.	✓ How can various graphical and algebraic methods be used to analyze functions? ✓ Why is it that none of the previously learned functions could model exponential growth or decay? ✓ How can transformations to a function rule and resulting effects on the graph be explained numerically? ✓ How does an understanding of linear systems help in solving non-linear systems?

CCSS	KNOWLEDGE	SKILLS
Modeling & Graphs of Functions F.IF.7 A.CED.2 F.IF.4 F.IF.5 F.IF.6 F.IF.8 F.IF.9 F.BF.1 F.BF.4	**Students will know that:** ● The graph of an equation in two variables is a visual representation of all possible solutions to the equation. ● The key features of the following types of functions can be determined from graphical, numerical, or algebraic representations: o Square root o Cube root o Absolute Value o Piecewise defined o Step ● The key features of functions are: o Intercepts o Intervals where the function is increasing, decreasing, positive, or negative	**Students will be able to:** ● Create equations in two or more variables to represent non-linear relationships between quantities. o Graph the equations on coordinate axes with labels and scales. ● Graph functions expressed symbolically. ● Sketch graphs given a verbal description of the relationship. o Show key features of the graph: ▪ By hand in simple cases ▪ Using technology for complex situations. o Interpret key features of graphs and tables in terms of the quantities. ● Compare the key features of two functions each represented in different ways.

	○ Relative maximums and minimums ○ Symmetries ○ Odd/Even Functions ○ End behavior ○ Periodicity	○ *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
	● The average rate of change can be interpreted as the slope between two points on the graph of a function for a given interval. ○ For non-linear functions, the average rate of change varies based on the interval given. ○ $\text{Average Rate of Change} = \frac{f(b)-f(a)}{b-a}$ for the interval $[a,b]$	● Calculate and interpret the average rate of change of a function over a specified interval algebraically, numerically and graphically.
	● Different forms of a function reveal information about the function. ○ Sometimes there is a need to write equivalent forms of a function based on the information required.	● Write a function defined by an expression in different but equivalent forms. ○ *For example, changing a quadratic function from standard form to vertex form.*
	● Domain restrictions on a function must be considered when applying functions to real world problems.	● Identify a reasonable domain for a function given its graph or the situation it describes. ○ *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then positive integers would be an appropriate domain for the function.*

	• Parts of a function have meaning and sometimes can be understood as separate functions combined together. • A function has an inverse that is a function if the relationship between inputs and outputs is one to one. Meaning: o Each input is paired with exactly one output and each output is paired with exactly one input.	• Write a function that describes a relationship between two quantities. o Combine standard function types using arithmetic operations. ▪ *For example, the cooling of an object over time can be modeled by combining a constant function and a decaying exponential function.* • Write an expression for an inverse function.
Exponential & Logarithmic Functions F.IF.7 F.LE.4	**Students will know that:** • Exponential functions are in the form: o $f(x) = ab^x$ where $b \neq 0$	**Students will be able to:** • Create equations in one variable and use them to solve problems involving exponential functions.
	• The solution to an exponential equation can be found using the properties of logarithms. o The solution to $ab^x = d$ can be represented as $x = \log_b\left(\frac{d}{a}\right)$ o *a and d are numbers and the base, b is 2, 10, or* e.	• Use their understanding of the relationship between exponential functions and logarithms to solve exponential equations. o *Evaluate the logarithm using technology.*
	• The key features of functions are: o Intercepts o End behavior	• Graph exponential and logarithmic functions by hand in the simple cases and using technology for more complicated cases.

<table>
<tr><td>Transformations on Functions

F.BF.3</td><td>Students will know that:

● Given a function $f(x)$, each of the following manipulations to the function results in a physical transformation to the shape of the graph:
o $f(x)+k$
o $f(x+k)$
o $k \cdot f(x)$
o $f(kx)$
o k can be any positive or negative real number.</td><td>Students will be able to:

● Experiment through the use of graphing technology to come to conclusions about transformations on functions.
● Identify the effects on the graph of a function based on algebraic manipulations of the function rule.</td></tr>
<tr><td rowspan="2">Non-linear Systems and Linear Programming

A.REI.11
A.CED3</td><td>Students will know that:

● The x-coordinates of the point(s) where the graphs of the equations y = f(x) and y = g(x) intersect are the solution(s) of the equation f(x) = g(x). Including cases where the functions are:
o Linear, Polynomial, Rational, Absolute value, Exponential, or Logarithmic</td><td>Students will be able to:

● Approximate the solutions of systems of non-linear functions by:
o Using technology to graph the functions
o Making tables of values
o Finding successive approximations.</td></tr>
<tr><td>● Knowledge of systems of equations/inequalities can aid in making decisions when solving complex problems.
● Sometimes the solutions to a system of equations/inequalities are not viable answers to the problem in context.</td><td>● Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</td></tr>
<tr><td></td><td colspan="2">Critical Vocabulary: Functions (Linear, Polynomial, Absolute Value, Exponential, Logarithmic, Piecewise Defined, Step, Inverse), Constraints, Systems of Equations, Formulas, Domain, Average Rate of Change, Interval, Equivalent</td></tr>
<tr><td colspan="3">Common 3rd Quarter Assessment</td></tr>
</table>

Pacing Chart
UNIT 3: Modeling with Functions

TIME FRAME	TOPIC	SUGGESTED PERFORMANCE TASKS ACTIVITIES/PROJECTS ASSESSMENTS	RESOURCES/INTERDISCIPLINARY CONNECTIONS
Middle of February until the Middle of March	Modeling & Graphs of Functions	Tutorial on Functions and Models. Talks about Cost, revenue and profit http://www.zweigmedia.com/RealWorld/tutorialsf0/framesF2A.html The Graph of a Function http://www.intmath.com/functions-and-graphs/4-graph-of-function.php Discusses the graphing of some more complex functions http://www.analyzemath.com/Graphing.html Graphing tool for modeling functions http://hs-mathematics.wikispaces.com/Rational+Functions Linear and Nonlinear Functions http://www.glencoe.com/sec/math/prealg/prealg05/study_guide/pdfs/prealg_pssg_G112.pdf YouTube video comparing linear and nonlinear functions http://www.youtube.com/watch?v=_850e1mEiD4 Discusses the reasonableness of the domain http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-147s.html Average rate of change http://www.mesacc.edu/~marfv02121/readings/average/index.html	Text Sections: 2-1, 2-6, 2-8, 3-4, 3-7 Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com

Mid March until end of March	Exponential & Logarithmic Functions	Teacher Resources: Chapter 2 Project #3, Chapter 9 Project #2 Discusses the graphing of some more exponential and logarithmic functions http://www.analyzemath.com/Graphing.html Various web-pages that support exponential and logarithmic functions http://www.intmath.com/exponential-logarithmic-functions/exponential-log-functions-intro.php	Text Sections: 2-4, 2-5, 9-3, 9-4, 9-5, 9-6, Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com
End of March until 1st week of April	Transformations on Functions	Transforming Functions http://regentsprep.org/Regents/math/algtrig/ATP9/funclesson1.htm	Text Sections: 3-1, 3-2 Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com
Mid April until the end of April	Non-linear Systems and Linear Programming	Nonlinear Systems http://www.dummies.com/how-to/content/how-to-solve-nonlinear-systems.html Examples and visuals of linear programming http://www.purplemath.com/modules/linprog3.htm	Text Sections: (Not Supported by current textbook) Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com

UNIT 4: Inferences and Conclusions from Data

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
✓ Normal distributions model many common, natural occurring phenomena.	✓ Why do you think measurements with a bell-shaped distribution are so common in the world?
✓ The way that data is collected determines the scope and nature of the conclusions that can be drawn from the data. ✓ Good statistical information about a population can be attained by studying a sample of the population.	✓ What are the purposes of and differences among sample surveys, experiments, and observational studies? ✓ How can you ensure that a sample of a population is good enough to represent the population from which it was taken?
✓ Probability models can be used to analyze situations and make fair decisions.	✓ How does the theoretical probability of an even occurring relate to the occurrence of the event? ✓ How can probability be used to make decisions fairly?

CCSS	KNOWLEDGE	SKILLS
Statistical Methods S-ID4 S-IC1 S-IC2	**Students will know that:** ● Statistics is a collection of procedures and principles for gathering data and analyzing information in order to help people make decisions when faced with uncertainty. ● The mean is the average of the data and the standard deviation is a measure of the spread of the data. o Mean for a sample is: $\bar{x} = \frac{\sum x_i}{n}$ o Standard deviation for a sample is: $s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$ or $s = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n-1}}$ ● The following approximations can be made for a data set that is normally distributed: o 68% of the values fall within 1 standard deviation of the mean in either direction.	**Students will be able to:** ● Make inferences about population parameters based on a random sample from that population. ● Use the mean and standard deviation of a data set to fit it to a normal distribution o Use the normal distribution to estimate population percentages. o Recognize that there are data sets for which this procedure is not appropriate. o Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

	○ 95% of the values fall within 2 standard deviation of the mean in either direction. ○ 99.7% of the values fall within 3 standard deviation of the mean in either direction.	
Data Analysis S-IC2 S-IC3 S-IC4 S-IC5 S-IC6	**Students will know that:** ● Data can be collected in different ways: ○ Sample surveys ○ Experiments ○ Observational study ○ Simulations	**Students will be able to:** ● Make inferences and justify conclusions from sample surveys, experiments, and observational studies. ● Recognize the purposes of and differences among sample surveys, experiments, and observational studies ○ Explain how randomization relates to each. ● Use data from a sample survey to estimate a population mean or proportion. ● Develop a margin of error through the use of simulation models for random sampling.
	● There is variability in experiments. Statistics is used as a way of dealing with, not eliminating, inherent randomness.	● Use data from a randomized experiment to compare two treatments ○ Use simulations to decide if differences between parameters are significant. ● Decide if a specified model is consistent with results from a given data-generating process. ○ *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* ● Compare theoretical and experimental results to evaluate the effectiveness of a treatment. ● Evaluate reports based on data.

Probability S-MD6 S-MD7	**Students will know that:** • (+) The probability of an event is the chance that an event will occur and is represented by a decimal, fraction or percent with a value that falls between 0 and 1. $\text{Probability} = \frac{\text{The number of ways an event can occur}}{\text{Total number of possible outcomes}}$ • (+) The relative frequency can be used as a measure of probability when an event occurs a large number of times. $\text{Relative Frequency} = \frac{\text{Total number of times an event actually occurs}}{\text{Total number of trials}}$	**Students will be able to:** • (+) Extend their knowledge of probability to more complex probability models. *For example, situations such as those involving quality control, or diagnostic tests that yields both false positive and false negative results.* • (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). • (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game)
	Critical Vocabulary: Statistics, Inferences, Population, Random Sampling, Mean, Standard Deviation, Normal Distribution, Data Sets, Normal Curve, Experiment, Simulation, Theoretical, Empirical, Sample Survey, Observational Study, Randomization, Margin of Error, Treatment, Parameters, (+) Fair Decisions	
Common Final Exam		

Pacing Chart

UNIT 4: Inferences and Conclusions from Data

TIME FRAME	TOPIC	SUGGESTED PERFORMANCE TASKS ACTIVITIES/PROJECTS ASSESSMENTS	RESOURCES/INTERDISCIPLINARY CONNECTIONS
End of April until the middle of May	Statistical Methods	Teacher Resources: Chapter 1 Project #3, 3 websites that explain standard deviation http://www.youtube.com/watch?v=HvDqbzu0i0E http://mathworld.wolfram.com/StandardDeviation.html http://davidmlane.com/hyperstat/A16252.html Video on standard deviation http://www.youtube.com/watch?v=Y2wnchUkTyQ Normal Distribution http://stattrek.com/Lesson2/Normal.aspx	Text Sections: 1-2, 1-6, 1-7, 1-8, 10-7, 11-1 to 11-3, 11-6, 11-7 Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com
Middle of May until Early June	Data Analysis	Teacher Resources: Chapter 1 Project #5, Chapter 11 #1, #2, #4 3 websites on data collection http://nnlm.gov/evaluation/workshops/measuring_your_impact/DataCollectionHandout.pdf http://dstraub.cis.gsu.edu:88/quant/4datacoll.asp http://www.prm.nau.edu/prm447/methods_of_data_collection_lesson.htm Simulations http://classroom.jc-schools.net/basic/math-prob.html	Text Sections: 1-1, 6-7, 3-9, 11-4, 11-5 Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com www.teachertube.com exchange.smarttech.com
Early June until End of year	Probability	Teacher Resources: Chapter 6 Project #1, #5 Probability and experiments http://www.mathsisfun.com/data/probability.html Simulations & probability	Text Sections: 6-1 to 6-4, 6-6, 6-8, 10-4 to 10-8 Teacher resources have activities, videos, projects, & enrichment. www.khanacademy.com

		http://classroom.jc-schools.net/basic/math-prob.html Experimental probability http://www.shodor.org/interactivate/activities/ExpProbability/ 2 websites on conditional probability http://www.mathgoodies.com/lessons/vol6/conditional.html http://www.mathgoodies.com/lessons/vol6/intro_probability.html	www.teachertube.com exchange.smarttech.com